The Kaon-Photoproduction Of Nucleons In The Chiral Quark Model

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Abstract

In this paper, we develop a general framework to study the meson-photoproductions of nucleons in the chiral quark model. The S and U channel resonance contributions are expressed in terms of the Chew-Goldberger-Low-Nambu (CGLN) amplitudes. The kaon-photoproduction processes, $\gamma p \to K^+\Lambda$, $\gamma p \to K^+\Sigma^0$, and $\gamma p \to K^0\Sigma^+$, are calculated. The initial results show that the quark model provides a much improved description of the reaction mechanism for the kaon-photoproductions of the nucleon with less parameters than the traditional phenomenological approaches.

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1. Introduction

The meson photoproduction of nucleons has always been a very important field to study the structure of hadrons. It was the investigations by Copley, Karl and Obryk[1], and Feynman, Kisslinger and Ravndal[2] in the pion-photoproduction 26 years ago that presented first evidences of underlying $SU(6) \otimes O(3)$ symmetry for the baryon structure in the quark model. The interest in this field has been recently revived by the establishment of the new experimental facilities such as CEBAF. In this paper, we present a new framework based on the chiral quark model to study the meson-photoproductions of nucleons, in particular the Kaon photoproductions. Some of the highlights of this project are

1. The transition amplitudes are chiral invariant in the low energy limit. Thus, the low energy theorem[3] in the threshold pion photoproduction is automatically recovered[4].

2. It provides an unified description for all s- and u- channel resonance contributions with only one set of parameters, which includes the constituent quark masses and the coupling constant between the Goldstone bosons, such as K, η and π, and quarks.

3. The contributions from the s and u channel resonances are expressed in terms of the CGLN[3] amplitudes so that the differential cross section and various polarizations in meson photoproductions can be easily carried out as the kinematics for the CGLN amplitudes is well known[5].

4. The form factors generated by the spatial integral lead to small cross sections in high energy limit, which is consistent with the data. This has been a major problem in the traditional hadronic models[6, 7].

5. It provides much improved description for the processes $\gamma p \to K^+\Sigma^0$ and $\gamma p \to K^0\Sigma^+$, while the results in hadronic models[8] show a factor 10 to 100 larger cross section for $\gamma p \to K^0\Sigma^+$ than that for $\gamma p \to K^+\Sigma^0$.

This project is an extension of our early investigation on the threshold pion photoproduction[4], in which the long-standing problem[9] of the low energy theorem in the threshold pion-photoproduction was clarified. Indeed, although the quark model does give good descriptions of the electromagnetic and strong decays of baryon resonances, it does not guarrantee the successes in the meson photoproductions. The key is that one has to combine the phenomenological quark model with chiral symmetry[10]. Moreover, since a baryon is being

treated as a three quark system in the quark model, the separation of the center of mass motion from the internal motion is crucial to recover the low energy theorem, this has been discussed in detail in the Compton scattering $\gamma N \to \gamma N$[11] and the threshold pion-photoproduction[4].

In section 2, we present the framework for the Kaon-photoproductions starting from the QCD Langrangian in the low energy region by assuming the Kaon as a Goldstone boson. Of course, there would be chiral symmetry breaking in Kaon productions. We shall only limit ourself in the symmetry limit and in the same time, treat the coupling constant as a free parameter. In section 3, we will present the formula for the leading order Born amplitudes that include the seagul term, the nucleon exchange in the s-channel, the Λ and Σ exchange in the u-channel, and the kaon and K^* exchange in the t-channel. In section 4, the u-channel contributions from the excited strange baryons are presented, in particular, the Σ^* resonances. In section 5, we show how to write the contributions from the nonstrange baryon resonances in terms of the CGLN amplitudes. In section 6, we present our numerical evaluations of the differential cross section, polarizations, and the total cross section for $\gamma p \to K^+\Lambda$, $\gamma p \to K^+\Sigma^0$ and $\gamma p \to K^0\Sigma^+$. Considering that many effects have not been included in our calculation and no serious effort is being made to fit the experimental data, the success of our approach is indeed very encouraging. In section 7, we shall discuss some effects that have been left in our calculation, and are important for the future investigations. Although only the kaon transition amplitudes are presented here, it should be pointed out that this approach is more general; the η and π photoproductions of the nucleon can be related to the Kaon photoproduction by the $SU(3)$ coefficients in each term. It is our hope that this approach could provide a unified description of the meson photoproductions of nucleons.

2. General Framework

In the pure chiral symmetry limit, the low energy QCD Langrangian can be written as[10]

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu(i\partial^\mu + V^\mu + A^\mu) - m\right]\psi + \ldots \tag{1}$$

where the vector and axial currents are

$$\begin{aligned} V_\mu &= \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right), \\ A_\mu &= i\frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right), \\ \xi &= e^{i\pi/f} \end{aligned} \tag{2}$$

f is a decay constant, and the field π can be written in terms of $SU(3)\otimes SU(3)$ Goldstone boson field. The gauge transformation of the axial vector A_μ in Eq. 2 leads to a quark-photon-kaon vertex;

$$H_{K,e} = \sum_j \frac{e_K}{f_K}\phi_K \bar{\psi}_j(s)\gamma_\mu^j\gamma_5^j\psi_j(u)A^\mu(\mathbf{k},\mathbf{r}_j), \tag{3}$$

where $A^\mu(\mathbf{k},\mathbf{r}_j)$ is the electromagnetic field, which generates a seagull term for the charge kaon production. The kaon-quark coupling at the tree level is therefore the standard pseudovector coupling

$$H_K = \sum_j \frac{1}{f_K}\bar{\psi}_j(s)\gamma_\mu^j\gamma_5^j\psi_j(u)\partial^\mu\phi_K \tag{4}$$

and the electromagnetic coupling is

$$H_e = -\sum_j e_j\gamma_\mu^j A^\mu(\mathbf{k},\mathbf{r}). \tag{5}$$

Generally, one can write the transition matrix element as

$$\begin{aligned}\mathcal{M}_{fi} = \langle N_f|H_{K,e}|N_i\rangle + \sum_j \Bigg\{ &\frac{\langle N_f|H_K|N_j\rangle\langle N_j|H_e|N_i\rangle}{E_i+\omega-E_j} \\ &+ \frac{\langle N_f|H_e|N_j\rangle\langle N_j|H_K|N_i\rangle}{E_i-\omega_K-E_j}\Bigg\} + \mathcal{M}_T\end{aligned} \tag{6}$$

where $N_i(N_f)$ is the initial (final) state of the nucleon, and $\omega(\omega_\pi)$ represents the energy of incoming (outgoing) photons(pions). The first term in Eq. 6 corresponds to the seagull diagram, which is a direct consequence of the chiral transformations, the second term corresponds to the S-channel resonance contribution which will be discussed in detail in section 5, the third is the U-channel resonance contributions; they come from the strange baryon resonances for the kaon photoproduction, and the last term $\mathcal{M}_T$ is the T-channel meson exchange contribution. As the couplings between the excited strange mesons and the nucleon are not well known, we will treat them as the free parameters, and in addition to the kaon exchange, only the K^* exchange will be included since the inclusion of the other excited meson exchanges leads additional free parameters to fit to the data.

The nonrelativistic expansion of the quark-photon-kaon interaction gives

$$H^{nr}_{K,e} = i\sum_j \frac{e}{f_K}a_j^\dagger(s)a_j(u)\boldsymbol{\sigma}_j\cdot\boldsymbol{\epsilon}, \tag{7}$$

where $\boldsymbol{\epsilon}$ is the polarization vector of photons, where $a_j^\dagger(s)$ and $a_j(u)$ is the creation and annihilation operator for the strange and up quarks for the charged kaon, while this term vanishes for the K^0 productions in the symmetry limit. Note that

$$\langle N_f|\sum_j a_j^\dagger(s)a_j(u)\boldsymbol{\sigma}_j|N_i\rangle = g_A\langle N_f|\boldsymbol{\sigma}|N_i\rangle, \tag{8}$$

where $\boldsymbol{\sigma}$ are the total spin operators of the nucleon, and g_A is the axial coupling constant for the kaon-nucleon-strange baryon couplings. In the $SU(6)\otimes O(3)$ symmetry limit, we have

$$g_A = \begin{cases} \sqrt{\frac{3}{2}} & \text{for } KP\Lambda \\ -\frac{1}{3\sqrt{2}} & \text{for } KP\Sigma^0 \end{cases}, \tag{9}$$

this leads to 1 and $-\frac{1}{3}$ ratios between the couplings of axial vector and vector for the Λ and Σ states. Thus, we have an expression for the seagull diagram

$$\langle N_f|H^{nr}_{K,e}|N_i\rangle = \frac{g_A e}{f_K}\langle N_f|\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}|N_i\rangle F(\mathbf{k},\mathbf{q}). \tag{10}$$

where $F(\mathbf{k},\mathbf{q})$ is the form factor and the function of the incoming photon and outgoing kaon momenta $\mathbf{k}$ and $\mathbf{q}$.

The differential cross section for the kaon-photoproduction in the center of mass frame is

$$\frac{d\sigma^{c.m.}}{d\Omega} = \frac{\alpha_e\alpha_K(E_N+M_N)(E_S+M_S)}{4s(M_S+M_N)^2}\frac{|\mathbf{q}|}{|\mathbf{k}|}|\mathcal{M}'_{fi}|^2 \tag{11}$$

where the factor eg_A/f_K has been removed from the transition matrix elements $\mathcal{M}'_{fi}$ so that it becomes dimensionless, and $\sqrt{s} = E_N+\omega_\gamma = E_S+\omega_K$ is the total energy in the c.m. frame. The coupling constant α_K is related to the factor g_A/f_K by the generalized Goldberg-Treiman relation[12], however, the quark mass effects lead to about 30 percent deviation from the measured value, while the Goldberg-Trieman relation is accurate within 5 percent for the pion couplings[13]. Therefore, the coupling α_K will be treated as a free parameter at present stage.

Generally, it is more convenient to study the meson-photoproductions of nucleon in terms of the CGLN amplitude[3];

$$\mathcal{M}'_{fi} = \mathbf{J}\cdot\boldsymbol{\epsilon} \tag{12}$$

where $\boldsymbol{\epsilon}$ is the polarization vector, and the current J is written as

$$\mathbf{J} = f_1\boldsymbol{\sigma} + if_2\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\mathbf{k}\times\boldsymbol{\sigma})}{|\mathbf{q}||\mathbf{k}|} + f_3\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{|\mathbf{q}||\mathbf{k}|}\mathbf{q} + f_4\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{\mathbf{q}^2}\mathbf{q} \tag{13}$$

in the center mass frame. The differential cross section in terms of the CGLN amplitude is[5]

$$
\begin{aligned}
|\mathcal{M}'_{fi}|^2 = \quad & Re \quad \Big\{ |f_1|^2 + |f_2|^2 - 2\cos(\theta) f_2 f_1^* \\
+ \quad & \frac{\sin^2(\theta)}{2} \left[|f_3|^2 + |f_4|^2 + 2 f_4 f_1^* + 2 f_3 f_2^* + 2\cos(\theta) f_4 f_3^* \right] \Big\} \qquad (14)
\end{aligned}
$$

where θ is the angle between the incoming photon momentum $\mathbf{k}$ and outgoing kaon momentum $\mathbf{q}$ in the center mass frame. The various polarization observables can also be expressed in terms of CGLN amplitudes, which can be found in Ref. [5].

The calculation of the S- and U- channel resonances contributions follows a procedure similar to that used in Compton scattering ($\gamma N \to \gamma N$)[11]. However, since our investigation is not limited to the low energy region, the relativistic kinematics should be used for the transition operator corresponding the center of mass motion; for example, the nonrelativistic operator $\frac{\mathbf{P}}{2M_T}$ should be replaced by $\frac{\mathbf{P}}{E+M_T}$ where $\mathbf{P}$ and E are the momentum and energy of the initial nucleon or the final strange baryon. Replacing the spinor $\bar{\psi}$ by $\psi^\dagger$ so that the γ matrices are replaced by the matrix $\boldsymbol{\alpha}$, the matrix elements for the electromagnetic interaction H_e can be written as

$$
\begin{aligned}
\langle N_j|H_e|N_i\rangle &= \langle N_j|\sum_j e_j \boldsymbol{\alpha}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}|N_i\rangle \\
&= i\langle N_j|[\hat{H}, \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}] - \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon}\boldsymbol{\alpha}_j \cdot \hat{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_j}|N_i\rangle \\
&= i(E_j - E_i - \omega)\langle N_j|g_e|N_i\rangle + i\omega\langle N_j|h_e|N_i\rangle, \qquad (15)
\end{aligned}
$$

where

$$
\hat{H} = \sum_j (\boldsymbol{\alpha}_j \cdot \mathbf{p}_j + \beta_j m_j) + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r_j}) \qquad (16)
$$

is the Hamiltonian for the composite system,

$$
g_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{i\mathbf{k}\cdot\mathbf{r}_j}, \qquad (17)
$$

$$
h_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon}(1 - \boldsymbol{\alpha}_j \cdot \hat{\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}_j}, \qquad (18)
$$

and $\hat{\mathbf{k}} = \frac{\mathbf{k}}{\omega_\gamma}$. Similarly, we have

$$
\langle N_f|H_e|N_j\rangle = i(E_f - E_j - \omega)\langle N_f|g_e|N_j\rangle + i\omega_\gamma\langle N_f|h_e|N_j\rangle. \qquad (19)
$$

Therefore the second and the third terms in Eq. 6 can be written as

$$\mathcal{M}'_{23} = i\langle N_f|[g_e, H_K]|N_i\rangle + i\omega \sum_j \left\{ \frac{\langle N_f|H_K|N_j\rangle\langle N_j|h_e|N_i\rangle}{E_i+\omega-E_j} \right. \\ \left. + \frac{\langle N_f|h_e|N_j\rangle\langle N_j|H_K|N_i\rangle}{E_i-\omega_K-E_j} \right\}. \tag{20}$$

The nonrelativistic expansion for h_e in Eq. 20 is[11, 4]

$$h_e = \sum_j \left[e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon}(1 - \frac{\mathbf{p}_j\cdot\mathbf{k}}{m_j\omega}) - \frac{e_j}{2m_j}\boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon}\times\hat{\mathbf{k}}) \right], \tag{21}$$

which h_e is only expanded to order $1/m$, and it has been shown[4] that the expansion to order $1/m_q$ is sufficient to reproduce the low energy theorem for the threshold pion-photoproductions[3]. The corresponding kaon-coupling for the initial nucleon and final Λ or Σ states is being written as

$$H_K^{nr} = \sum_j \left\{ \frac{\omega_K}{E_S+M_S}\boldsymbol{\sigma}_j\cdot\mathbf{P}_f + \frac{\omega_K}{E_N+M_S}\boldsymbol{\sigma}_j\cdot\mathbf{P}_i - \boldsymbol{\sigma}_j\cdot\mathbf{q} \right. \\ \left. + \frac{\omega_K}{2\mu_s}\boldsymbol{\sigma}_j\cdot\mathbf{p}_j \right\} \frac{a_j^\dagger(s)a_j(u)}{g_A} \tag{22}$$

where ω_K is the energy of the emitting kaons, and $\frac{1}{\mu_s} = \frac{1}{m_s} + \frac{1}{m_q}$. The first three terms in Eq. 22 corresponds to the center of mass motion, and the last term represents the internal transition. Similarly, the K^* coupling between the nucleon and strange baryon has the structure;

$$H_{K^*}^{nr} = g_{K^*} \sum_j \left\{ \frac{1}{E_S+M_S} P_S\cdot\epsilon_v + \frac{1}{E_N+M_S} P_N\cdot\epsilon_v \right. \\ \left. + \frac{1}{2m_j}\boldsymbol{\sigma}_j\cdot(\boldsymbol{\epsilon}_v\times\hat{\mathbf{k}}) \right\} \frac{a_j^\dagger(s)a_j(u)}{g_A}, \tag{23}$$

where $P_S\cdot\epsilon_v = E_S\epsilon_v^0 - \mathbf{P}_S\cdot\boldsymbol{\epsilon}_v$, and only the operator associated with the c.m. motion is presented.

3. The Leading Order Born Amplitudes

Following the same procedure in Ref [4], the amplitudes for the seagull term is

$$\mathcal{M}_s = F(\mathbf{k},\mathbf{q}) \left[1 + \frac{\omega_K}{2} \left(\frac{1}{E_N+M_N} + \frac{1}{E_S+M_S} \right) \right] \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}, \tag{24}$$

thus, the the seagull term only contribute to the CGLN amplitude f_1. If the outgoing kaon is being considered as point like particle, the form factor would be

$$F(\mathbf{k},\mathbf{q}) = exp\left(-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}\right) \tag{25}$$

in the harmonic oscillator basis, where α is the oscillator strength. One could use the quark pair creation model[14] to calculation this form factor for the finite size kaons, we find that Eq. 25 would be modified by the additional form factor that are function of $\mathbf{k}^2$ and $\mathbf{q}^2$, in the same time, the finite size kaon also destroys the chiral symmetry (see discussion in section 7).

The matrix element for the nucleon pole term is found to be

$$\mathcal{M}_N = -\omega_K e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}\left(\frac{1}{E_S+M_S}+\frac{1}{E_N+M_N}\right)\left(1+\frac{\mathbf{k}^2}{4P_N\cdot k}\mu_N\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$$
$$+ie^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\left[\frac{\omega_K}{2}\left(\frac{1}{E_S+M_S}+\frac{1}{E_N+M_N}\right)+1\right]\frac{\mu_N}{2P_N\cdot k}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) \tag{26}$$

where $P_N\cdot k = \omega_\gamma(E_N+\omega_\gamma)$, and μ_N is the magnetic moments of the nucleon.

The matrix elements for the U-channel Λ and Σ exchange term is

$$\mathcal{M}_{\Lambda\Sigma} = -e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}\frac{M_S}{2M_N}\left(\frac{\mu_S}{P_S\cdot k}+\frac{g_{\Lambda\Sigma}\mu_{\Lambda\Sigma}}{P_S\cdot k\pm\delta m^2}\right)$$
$$\left\{\frac{\omega_K\mathbf{k}^2}{2}\left(\frac{1}{E_S+M_S}+\frac{1}{E_N+M_N}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}-\right.$$
$$\left. i\left[\frac{\omega_K}{2}\left(\frac{1}{E_S+M_S}+\frac{1}{E_N+M_N}\right)+1\right]\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{q}\right\} \tag{27}$$

where

$$g_{\Lambda\Sigma} = \begin{cases} \frac{g_\Sigma}{g_\Lambda} & \text{for } \gamma p\to K^+\Lambda \\ \frac{g_\Lambda}{g_\Sigma} & \text{for } \gamma p\to K^+\Sigma^0 \\ 0 & \text{for } \gamma p\to K^0\Sigma^+ \end{cases} \tag{28}$$

is the ratio between the coupling constants for Λ and Σ final states, $\mu_{\Lambda\Sigma}=1.61$ is the magnetic moments for the transition between the Λ and Σ^0 states, and $P_S\cdot k = E_S\omega_\gamma+\mathbf{k}\cdot\mathbf{q}$, notice that the final baryon state has the total momentum $-\mathbf{q}$ in the center of mass system. If the Λ and Σ pole terms in the U-channel are replaced by the nucleon, and kaons replaced by pions, the low energy theorem in the threshold pion-photoproductions should be recovered by combining Eqs. 22, 26 and 27.

The matrix elements for the t-channel are

$$\mathcal{M}_K = e^{-\frac{\mathbf{k}^2}{8\beta^2}-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}}\frac{(M_S+M_N)\mathbf{q}\cdot\boldsymbol{\epsilon}}{q\cdot k}\left(\frac{1}{E_S+M_S}\boldsymbol{\sigma}\cdot\mathbf{q}-\frac{1}{E_N+M_N}\boldsymbol{\sigma}\cdot\mathbf{k}\right) \tag{29}$$

for the kaon exchange and

$$\mathcal{M}_{K^*} = e^{-\frac{\mathbf{k}^2}{8\beta^2}-\frac{(\mathbf{k}-\mathbf{q})^2}{6\alpha^2}} \frac{(M_{K^0}+M_{K^*})g_{K^*}}{t-M_{K^*}^2}\Bigg\{\frac{1}{2\mu_s}\boldsymbol{\sigma}\cdot((\mathbf{q}-\mathbf{k})\times(\mathbf{k}\times\boldsymbol{\epsilon}))$$
$$+i\frac{g_V}{g_A}\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})\left[\frac{1}{E_S+M_S}\left(1+\frac{M_S^2-M_N^2+t}{2M_{K^*}^2}\right)-\frac{t+M_N^2-M_S^2}{2M_{K^*}^2(E_N+M_N)}\right]\Bigg\} \quad (30)$$

for the K^* exchange, where $t = M_K^2 - 2(\omega_\gamma\omega_K - \mathbf{q}\cdot\mathbf{k})$. The parameter g_{K^*} in Eq. 30 is the ratio between the couplings of K^0 and K^* to the nucleon and the strange baryon.

4. The U-channel resonance contribution

The leading U-channel contribution comes from the resonance $\Sigma^*(1385)$. In the quark model, it belongs to the same **56** multiplet as the resonances Λ and Σ, and there should be no orbital excitations in the symmetry limit. Thus, only the c.m. motion part of transition operator in Eq. 22 would contribute. We can rewrite the transition operator corresponding to the c.m. mass motion in Eq. 22 as

$$H_K^c = \sum_j \boldsymbol{\sigma}_j \cdot \mathbf{A}\frac{a_j^\dagger(s)a_j(u)}{g_A} \quad (31)$$

where

$$\mathbf{A} = -\omega_K\left(\frac{1}{E_N+M_N}+\frac{1}{E_S+M_S}\right)\mathbf{k} - \left(\omega_K\frac{1}{E_S+M_S}+1\right)\mathbf{q}. \quad (32)$$

Notice that the initial nucleon and intermediate Σ states have the c.m. momenta $-\mathbf{k}$ and $-\mathbf{k}-\mathbf{q}$ respectively. Then the contribution from the $\Sigma(1385)$ can be written as

$$\mathcal{M}_{\Sigma(1385)} = \frac{M_S g_S e^{-\frac{\mathbf{q}^2+\mathbf{k}^2}{6\alpha^2}}}{M_N(P_S\cdot k+\delta M_{\Sigma^*}^2/2)}\left[i2\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right] \quad (33)$$

where the factor g_S is

$$g_S = \begin{cases} -\frac{\mu_\Lambda}{3} & \text{for } \gamma p \to K^+\Lambda \\ 4\mu_{\Sigma^0} & \text{for } \gamma p \to K^+\Sigma^0 \\ \frac{2\mu_{\Sigma^+}}{3} & \text{for } \gamma p \to K^0\Sigma^+ \end{cases}, \quad (34)$$

and $\delta M_{\Sigma^*}^2 = M_{\Sigma^*}^2 - M_S^2$. The magnetic moments in Eq. 34 are $\mu_\Lambda = -0.61$, $\mu_{\Sigma^0} \approx 1.0$ and $\mu_{\Sigma^+} = 2.38$.

For the excited resonances, we follow the procedure in Ref. [11]. In the harmonic oscillator basis, one can write the transition matrix elements for the excited strange baryon resonances as

$$\mathcal{M}_Y = \left(\mathcal{M}_Y^3 + \mathcal{M}_Y^2\right) e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}} \tag{35}$$

in the symmetry limit. The first term represents the process in which the incoming photon and outgoing kaon are absorbed and emitted by the same quarks, its general expression can be obtained by the second quantization approach in the harmonic oscillator basis[11];

$$\begin{aligned}\mathcal{M}_Y^3 = \frac{-1}{6m_s}\left[i\frac{g_V}{g_A}\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + \boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right] F(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, -P_S\cdot k) \\ +\frac{1}{18}\left[\frac{\omega_K\omega_\gamma}{\mu_s}\left(1+\frac{\omega_\gamma}{2m_s}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} + \frac{2}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right] F(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, -P_S\cdot k - \delta M^2) \\ +\frac{\omega_K\omega_r}{54\alpha^2\mu_s} F(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}, -P_S\cdot k - 2\delta M^2).\end{aligned} \tag{36}$$

The function $F(x,y)$ in Eq. 36 corresponds to the product of the spatial integral and the propagator for the excited states, it can be written as

$$F(x,y) = \sum_n \frac{M_S}{n!(y - n\delta M^2)} x^n, \tag{37}$$

where $n\delta M^2 = (M_n^2 - M^2)/2$ represents the mass difference between the ground state and excited states with the major quantum number n in the harmonic oscillator basis, which will be chosen as the average mass differences between the ground state and the negative parity baryons. Therefore, first term in Eq. 36 corresponds to the correlation between the magnetic transition and the c.m. motion of the kaon transition operator, it contributes to the leading Born terms in the U-channel. The second term in Eq. 36 is the correlations among the internal and c.m. motions of the photon and kaon transition operators, this term only contributes to the transitions between the ground and $n \geq 1$ excited states in the harmonic oscillator basis. The third term in Eq. 36 corresponds to the correlation of the internal motions between the photon and kaon transition operators, which only contributes to the transition between the ground and $n \geq 2$ excited states. In the $SU(6)$ symmetry limit, the ratio $\frac{g_V}{g_A}$ is 1 for the Λ final state and -3 for the Σ final state. The second term $\mathcal{M}_2$ in Eq. 35 represents the process in which the incoming photon and outgoing kaon are absorbed and emitted by different quarks, it can be written as

$$\frac{\mathcal{M}_Y^2}{g_S'} = \frac{1}{6m_q}\left[-ig_v'\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + g_a'\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right] F(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}, -P_S\cdot k)$$

$$
-\frac{1}{36}\left[\frac{\omega_K\omega_\gamma}{\mu_s}\left(1+g'_a\frac{\omega_\gamma}{2m_q}\right)+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]F(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},-P_S\cdot k-\delta M^2)
$$

$$
\frac{\omega_K\omega_\gamma}{216\alpha^2\mu_s}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}F(-\frac{\mathbf{k}\cdot\mathbf{q}}{6\alpha^2},-P_S\cdot k-2\delta M^2) \quad (38)
$$

where the factor g'_S equals to 1 for both charged kaon productions, 4 for the K^0 productions, the factors g'_v and g'_a are

$$
g'_v=\begin{cases} 1 & \text{for } \gamma p\to K^+\Lambda \\ -7 & \text{for } \gamma p\to K^+\Sigma^0 \\ 1 & \text{for } \gamma p\to K^0\Sigma^+ \end{cases} \quad (39)
$$

and

$$
g'_a=\begin{cases} 1 & \text{for } \gamma p\to K^+\Lambda \\ 9 & \text{for } \gamma p\to K^+\Sigma^0 \\ 0 & \text{for } \gamma p\to K^0\Sigma^+ \end{cases} . \quad (40)
$$

An interesting observation from these expressions is that the transition matrix elements $\mathcal{M}^3_Y$ and $\mathcal{M}^2_Y$ that correspond to the incoming photons and outgoing kaons being absorbed and emitted by the same and different quarks differ by a factor $\left(-\frac{1}{2}\right)^n$, thus the transition matrix element $\mathcal{M}^3_Y$ becomes dominant as the quantum number n increases.

Eqs. 35 and 38 can be summed up to any quantum number n, however, the excited states with large quantum number n become less significant for the U-channel resonance contributions. Thus, we only include the excited states with $n\leq 2$, which is the minimum number required for the contribution from every term in Eqs. 35 and 38.

5. The S-channel resonance contribution

There are two major components for the S-channel resonance contributions; the well known baryon resonances below 2 GeV that correspond to $n\leq 2$ according to the $SU(6)\otimes O(3)$ classification and the resonances above 2 GeV that are not well known both theoretically and experimentally. One could regard the resonances above 2 GeV as the continuum contributions, however, they must be included in the calculations if one intends to calculate the photoproductions above 2 GeV in the c.m. frame. The advantage of the quark model approach is that it provides us a unified description of the S- and U-channel resonance contributions and the continuum contributions by the same set of parameters.

For the S-channel resonance processes, the operator **A** in Eq. 31 should be

$$\mathbf{A} = -\left(\omega_K \frac{1}{E_S + M_S} + 1\right)\mathbf{q} \tag{41}$$

in the c.m. frame. The calculation of the S-channel resonance contributions is similar to that of the U-channel resonance contributions. However, since the operator **A** is only proportional to the final state momentum **q**, the partial wave analysis can be easily carried out for the S-channel resonances.

In general, one can write the S-channel resonance amplitudes as

$$\mathcal{M}_R = \frac{2}{s - M_R^2} e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}} \mathcal{O}_R, \tag{42}$$

where $\sqrt{s} = E_N + \omega_\gamma = E_S + \omega_K$ is the total energy of the system, and $\mathcal{O}_R$ is determined by the structure of each resonance. Eq. 42 shows that there should be a form factor, $e^{-\frac{\mathbf{k}^2+\mathbf{q}^2}{6\alpha^2}}$ in the harmonic oscillator basis, even in the real photon limit. This leads to a smaller cross section in the high energy limit, which has been a major problem in the hadronic models[7, 6]. If the mass of a resonance is above the threshold, the mass M_R in Eq. 42 should be changed to

$$M_R^2 \to M_R(M_R - i\Gamma(\mathbf{q})). \tag{43}$$

$\Gamma(\mathbf{q})$ in Eq. 43 is the total width of the resonance, and a function of the final state momentum **q**. For a resonance decay to a two body final state with orbital angular momentum l, the decay width $\Gamma(\mathbf{q})$ can be written as

$$\Gamma(\mathbf{q}) = \Gamma_R \left(\frac{|\mathbf{q}|}{|\mathbf{q}_R|}\right)^{2l+1} \frac{\sqrt{s}}{M_R} \frac{D_l(\mathbf{q})}{D_l(\mathbf{q}_R)}, \tag{44}$$

where $|\mathbf{q}_R| = \sqrt{\frac{(M_R^2 - M_S^2 + M_K^2)^2}{4M_R^2} - M_K^2}$ and Γ_R are the momentum of one final state and the decay width at the S-channel resonance mass M_R. The function $D_l(\mathbf{q})$ is called fission barrier[15], and wavefunction dependent; here we use

$$D_l(\mathbf{q}) = exp\left(-\frac{\mathbf{q}^2}{3\alpha^2}\right), \tag{45}$$

which is independent of l. A similar formula used in I=1 $\pi\pi$ and and p-wave $I = 1/2$ $K\pi$ scattering was found in excellent agreement with data in the ρ and K^* meson region[16]. Of course, whether Eq. 45 is suitable for the baryon resonance is unclear, and should be studied in the $P_{33}(1232)$ region of pion-photoproduction, where the data are well known.

The first S-channel resonance is the resonance $P_{33}(1232)$, the transition matrix element is similar to that of the resonance $\Sigma^*(1387)$ in U-channel, however, it does not contribute to the process $\gamma p \to K^+\Lambda$ due to the isospin couplings. The matrix element for the Σ final state is

$$\mathcal{O}_{P_{33}(1232)} = \mu_N g_\Delta \left[i2\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + \boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right], \tag{46}$$

where

$$g_\Delta = \begin{cases} \frac{4}{3} & \text{for } \gamma p \to K^+\Sigma^0 \\ -\frac{2}{3} & \text{for } \gamma p \to K^0\Sigma^+ \end{cases} \tag{47}$$

is determined by the $SU(6)$ symmetry. Any isospin 3/2 state in **56** representation should have the same factor g_Δ. It is straightforward to express the transition matrix element in Eq. 46 in terms of the CGLN amplitudes, and the multiple decomposition of the CGLN amplitudes (see Ref. [5]) shows that the transition matrix element in Eq. 46 has the characteristics of the M_1^+ transition.

For the P-wave baryon resonance with the quantum number $n = 1$, there are seven resonances which have been identified experimentally. However, the photon transitions from the proton target are simplified due to the Moorhouse selection rule[17]; the photoabsorption amplitudes for $\gamma p \to N^*$ belonging to $SU(3)$ octets with spin $\frac{3}{2}$ vanish for the transition operator in Eq. 21. This reduces the number of the resonances that contribute to the kaon photoproduction of the proton target to 4. Furthermore, the isospin couplings for $N^* \to K^+\Lambda$ only allow the resonances with isospin $\frac{1}{2}$, which reduces the number of the P-wave resonances to 2 for Λ final state. We find that the transition amplitude for the resonance with the quantum number $\mathbf{70}, N(^2P_M)\frac{1}{2}^-$ is

$$\mathcal{O}_{S_{11}} = g_M \frac{\omega_\gamma}{18}\left(\frac{3\omega_K}{2\mu_s} + \frac{1}{\alpha^2}\mathbf{q}\cdot\mathbf{A}\right)\left(1+\frac{|\mathbf{k}|}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} \tag{48}$$

where g_M factor is 1 for the Λ final state and -1 for both Σ^0 and Σ^+ final state. This transition matrix element is a pure CGLN amplitude f_1, and independent of the scattering angle since it is a S-wave. It is also a product of the amplitudes for $\gamma p \to S_{11}$ and kaon decays. The photoabsorption amplitude agrees with the result in Ref. [18], and the kaon decay amplitude is the same as the expression in Table 1 in Ref. [19] with $g - \frac{1}{3}h = \frac{|\mathbf{A}|}{|\mathbf{q}|}$, and $h = \frac{\omega_K}{2\mu_s}$. Note that $\mathbf{A}$ has a negative sign, this is consistent with the fitted value for $g - \frac{1}{3}h$ and h in Ref. [19]. There are two S_{11} resonances with masses around 1.6 GeV; $S_{11}(1535)$ and $S_{11}(1650)$. Traditionally, the resonance $S_{11}(1535)$ is being classified as a $\mathbf{70}, N(^2P_M)\frac{1}{2}^-$ state, however, the potential quark model

calculation[20] found that the two states, $\mathbf{70}N(^2P_M)\frac{1}{2}^-$ and $\mathbf{70}N(^4P_M)\frac{1}{2}^-$, are strongly mixed. Thus, we have

$$\begin{aligned}\mathcal{O}_{S_{11}(1535)} &= 0.6\mathcal{O}_{S_{11}} \\ \mathcal{O}_{S_{11}(1650)} &= 0.4\mathcal{O}_{S_{11}}\end{aligned} \tag{49}$$

where the coefficients 0.6 and 0.4 come from the mixing angle obtained from the potential model calculation.

The transition amplitude for the resonance $D_{13}(1520)$ is

$$\begin{aligned}\mathcal{O}_{D_{13}(1520)} = -g_M\frac{1}{12}\Big[&\frac{i\mathbf{q}\cdot\mathbf{k}}{m_q\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\frac{1}{3}\left(\frac{\mathbf{k}^2}{m_q\alpha^2}+\frac{2\omega_\gamma}{\alpha^2}\right)\mathbf{A}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} \\ &-\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\mathbf{q}\cdot\boldsymbol{\epsilon}\Big]\end{aligned} \tag{50}$$

where the factor g_M is the same as that in Eq. 48, since the states S_{11} and D_{13} belong to the same $SU(3)$ representation. Notice that there is little mixing between the spin 1/2 and spin 3/2 state for the resonance $D_{13}(1520)$[20].

If one treats the transition amplitudes $\mathcal{O}$ for the resonance S_{11} and D_{13} in Eqs. 48 and 50 as a function of the quark mass m_q, the transition amplitudes for the isospin 3/2 states can be related to those for the resonances S_{11} and D_{13}. The amplitudes for the resonances $S_{31}(1670)$ and $D_{33}(1700)$ are

$$\begin{aligned}\mathcal{O}_{S_{31}(1670)} &= 2\mathcal{O}_{S_{11}}(g_M\to g_{\Delta_M}, m_q\to -3m_q), \\ \mathcal{O}_{D_{33}(1700)} &= 2\mathcal{O}_{D_{13}}(g_M\to g_{\Delta_M}, m_q\to -3m_q)\end{aligned} \tag{51}$$

which replace the factor g_M in Eqs. 48 and 50 by g_{Δ_M}, and the quark mass m_q by $-3m_q$. The factor g_{Δ_M} is 1 for the charged kaon production and $-\frac{1}{2}$ for the K^0 production, and these two states do not couple to the Λ final state.

The situation for the positive parity baryon states with the quantum number $n=2$ is not as clear as the P-wave baryons. The well known resonances belong to the **56** multiplets. The transition amplitudes for the radial excitation states are

$$\mathcal{O}_{P_{11}(1440)} = -\frac{\mathbf{k}^2}{216\alpha^2}\left(\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}+\frac{\omega_K}{\mu_s}\right)i\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) \tag{52}$$

for the Roper resonance, and

$$\mathcal{O}_{P_{33}(1600)} = \frac{g_\Delta\mathbf{k}^2}{108m_q\alpha^2}\left(\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}+\frac{\omega_K}{\mu_s}\right)\Big[i2\mathbf{q}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\boldsymbol{\sigma}\cdot(\mathbf{q}\times(\boldsymbol{\epsilon}\times\mathbf{k})\Big] \tag{53}$$

for the resonance $P_{33}(1600)$, where the factor g_Δ is given in Eq. 47.

The transition amplitudes for the $N(^2D_S)$ states are

$$\mathcal{O}_{P_{13}(1720)} = \frac{\mathbf{k}^2}{90\alpha^2}\left(\frac{\mathbf{A}\cdot\mathbf{q}}{\alpha^2}+\frac{5\omega_K}{2\mu_s}\right)\left[\frac{i\omega_\gamma}{6m_q}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\right.$$
$$\left.+\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\mathbf{k}\cdot\mathbf{q}\left(1+\frac{\omega_\gamma}{2m_q}\right)+\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\epsilon}\right] \tag{54}$$

for the resonance $P_{13}(1720)$, and

$$\mathcal{O}_{F_{15}(1688)} = \frac{1}{36\alpha^4}\left\{\omega_\gamma^3\left[\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\mathbf{k}\cdot\mathbf{q}-\frac{1}{5}\mathbf{q}^2(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\mathbf{k}\cdot\mathbf{A}+\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{A})\right]\right.$$
$$\left.-\frac{1}{4m_q}\left[i\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})\left((\mathbf{k}\cdot\mathbf{q})^2-\frac{1}{5}\mathbf{q}^2\mathbf{k}^2\right)+\frac{2}{5}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\mathbf{k}\cdot\mathbf{q})^2\mathbf{k}\cdot\mathbf{A}\right]\right\} \tag{55}$$

for the resonance $F_{15}(1688)$. The isospin 3/2 states $\Delta(^4D_S)$ are concentrated around mass 1.9 GeV, therefore, we simply treat them as degenerate. The amplitude for the resonances $P_{31}(1910)$ and $P_{33}(1920)$ for the Σ final state gives

$$\mathcal{O}_{P_{3(1+3)}} = \frac{-g_\Delta\mathbf{k}^2}{108m_q\alpha^2}\left(\frac{2\mathbf{A}\cdot\mathbf{q}}{5\alpha^2}+\frac{\omega_K}{\mu_s}\right)\left[i2\mathbf{q}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\boldsymbol{\sigma}\cdot(\mathbf{q}\times(\boldsymbol{\epsilon}\times\mathbf{k})-3\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\mathbf{k}\cdot\mathbf{q}\right]. \tag{56}$$

The amplitude for the resonances $F_{35}(1905)$ and $F_{37}(1950)$ is

$$\mathcal{O}_{F_{3(5+7)}} = \frac{g_\Delta}{36m_q\alpha^4}\left[(i2\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right.$$
$$\left.\left((\mathbf{k}\cdot\mathbf{q})^2-\frac{1}{5}\mathbf{k}^2\mathbf{q}^2\right)+\frac{2}{5}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\mathbf{k}\cdot\mathbf{A}\mathbf{k}^2\mathbf{q}^2\right]. \tag{57}$$

These are the transition amplitudes for the **56** multiplet. The coefficients for the isospin 3/2 states in the **56** multiplet are significantly larger than those for the isospin 1/2 states for the Σ final states. One also finds that the contributions from the Roper resonance $P_{11}(1440)$ are small compare to the resonance $F_{15}(1688)$. However, the traditional quark model does not give a good description for the Roper resonances, and the gluonic degrees of freedom may play an explicit role in this resonance[21], the consequences from the gluonic degrees of freedom in the Roper resonance remain to be studied.

The contributions from those resonances belonging to the **70** multiplet can be related to the transition amplitudes of the **56** multiplets. The resonance $P_{11}(1710)$ which is being assigned as a $N(^2S_M)\frac{1}{2}^+$ state can be related to the Roper resonance by

$$\mathcal{O}_{P_{11}(1710)} = g_M\frac{1}{2}\mathcal{O}_{P_{11}(1440)}. \tag{58}$$

The contribution from the resonance $P_{31}(1750)$ is

$$\mathcal{O}_{P_{31}(1750)} = g_{\Delta_M}\frac{1}{3}\mathcal{O}_{P_{11}(1440)}. \tag{59}$$

Indeed, the electromagnetic coupling of this resonance is very weak, which was only seen in the πN scattering[22]. If we assume the resonance $F_{15}(2000)$ as a $\mathbf{70}N(^2D_M)\frac{5}{2}^+$ state, its transition amplitude is related to that of the resonance $F_{15}(1688)$ by

$$\mathcal{O}_{F_{15}(2000)} = g_M\frac{1}{2}\mathcal{O}_{F_{15}(1688)}. \tag{60}$$

These relations are determined by the $SU(6)\otimes O(3)$ symmetry. One could easily obtain the transition amplitudes for other **70** multiplet states, we will neglect those resonance here since their couplings are very weak from our calculation, which do not make much difference even if they are included, and their width and masses have not been determined experimentally.

The transition amplitudes from the S-channel resonances provide us some important insights into the role of the baryon resonances in the kaon photoproduction even without further numerical evaluation; the resonances of the higher partial waves are very important in the process $\gamma p\to K^+\Lambda$, in particular the resonance $D_{13}(1520)$ of P-wave baryons and the resonance $F_{15}(1688)$ of $n=2$ baryon state, which are usually neglected in the traditional hadronic model. For the processes $\gamma p\to K^+\Sigma^0$ and $\gamma p\to K^0\Sigma^+$, the contributions from the isospin 3/2 states, in particular those resonances in **56** multiplet, are dominant. Therefore, the processes $\gamma p\to K^+\Sigma^0$ and $\gamma p\to K^0\Sigma^+$ provide us a very important probe to the resonances with isospin 3/2, a particular example is the resonances $F_{37}(1950)$, $F_{35}(1905)$, $P_{33}(1920)$ and $P_{31}(1910)$.

If one intends to calculate the reaction beyond 2 GeV in the center of mass frame, the higher resonances with quantum number $n=3$ and $n=4$ must be included. There is a little knowledge of resonances in these region except high partial wave resonances. However, we can assume that the resonances for $n\geq 2$ are degenerate, the sum of the transition amplitudes from these resonances can be obtained through the approach in Ref. [11]. The transition amplitude for the nth harmonic oscillator shell is

$$\mathcal{O}_n = \mathcal{O}_n^2 + \mathcal{O}_n^3 \tag{61}$$

where the amplitudes $\mathcal{O}_n^2$ and $\mathcal{O}_n^3$ have the same meaning as the amplitudes $\mathcal{M}_Y^2$ and $\mathcal{M}_Y^3$ in Eqs. 36 and 38, and we have

$$\frac{\mathcal{O}_n^3}{g_e} = \frac{1}{6m_q}\left[i\frac{g_V}{g_A}\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k}) + \boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k}))\right]\frac{1}{n!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^n$$

$$+\frac{1}{18}\left[\frac{\omega_K\omega_\gamma}{\mu_s}\left(1+\frac{\omega_\gamma}{2m_q}\right)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}+\frac{2}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]\frac{1}{(n-1)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-1}$$
$$+\frac{\omega_K\omega_\gamma}{54\alpha^2\mu_s}\frac{1}{(n-2)!}\left(\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}\right)^{n-2} \tag{62}$$

and

$$\frac{\mathcal{O}_n^2}{g'_S}=\frac{1}{6m_q}\left[-ig'_v\mathbf{A}\cdot(\boldsymbol{\epsilon}\times\mathbf{k})+g'_a\boldsymbol{\sigma}\cdot(\mathbf{A}\times(\boldsymbol{\epsilon}\times\mathbf{k})\right]\frac{1}{n!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^n$$
$$-\frac{1}{36}\left[\frac{\omega_K\omega_\gamma}{\mu_s}\left(1+g'_a\frac{\omega_\gamma}{2m_q}\right)+\frac{2\omega_\gamma}{\alpha^2}\boldsymbol{\sigma}\cdot\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{q}\right]\frac{1}{(n-1)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-1}$$
$$\frac{\omega_K\omega_\gamma}{216\alpha^2\mu_s}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q}\frac{1}{(n-2)!}\left(\frac{-\mathbf{k}\cdot\mathbf{q}}{6\alpha^2}\right)^{n-2} \tag{63}$$

where g'_S is the same as in Eq. 38, g'_v and g'_a are given in Eqs. 39 and 40, and g_e is 2 for charged kaons and -1 for the neutral kaons. Generally, the resonances with larger quantum number n become important as the energy increases. Note that the amplitude $\mathcal{O}_n^2$ generally differs from the amplitude $\mathcal{O}_n^3$ by a factor of $\left(-\frac{1}{2}\right)^n$, this shows that the process that the incoming photon and outgoing kaon are absorbed and emitted by the same quark becomes more and more dominant as the energy increases. Furthermore, the resonances with partial wave $l=n$ become dominant, of which the isospin is $1/2$ for $\gamma p\to K^+\Lambda$ and $3/2$ for $\gamma p\to K\Sigma$. Thus, we could use the mass and decay width of the high spin states in Eq. 43; the resonance $G_{17}(2190)$ for the $n=3$ states and the resonance $H_{19}(2220)$ for the $n=4$ states in $\gamma p\to K^+\Lambda$, and the resonance $G_{37}(2200)$ for the $n=3$ states and the resonance $H_{37}(2420)$ for n=4 states in $\gamma p\to K\Sigma$. Indeed, only the couplings for the high spin states are strong enough to be seen experimentally, this is consistent with the conclusions of the quark model

6. The Numerical evaluation

It is straightforward to express every term in section 3, 4 and 5 in terms of the CGLN amplitudes so that the differential cross section and various polarization could be easily carried out. However, it should be pointed out that the nonrelativistic wavefunction is no longer adequate to describe the meson photoproduction processes because the relativistic effects become more and more important as the energy increases. A practical way to correct this shortcoming is the introduction of the Lorentz boost factor[23], which was

used in the calculations of the transition amplitudes as a function of Q^2 in the electroproduction[24]. We shall adopt a similar procedure in the meson-photoproduction for the transition amplitudes as a function of the energies of initial and final state, the CGLN amplitudes become

$$f_i(\mathbf{k},\mathbf{q}) \rightarrow \frac{M_S M_N}{E_S E_N} f_i(\frac{M_N}{E_N}\mathbf{k}, \frac{M_S}{E_S}\mathbf{q}) \tag{64}$$

where $i = 1 \ldots 4$, and $\frac{M_N}{E_N}$ $(\frac{M_S}{E_S})$ is a Lorentz boost factor for the initial (final) state.

The parameters used in our calculations have standard values except the coupling constants α_K and g_{K^*}, which will be fitted to the experimental data. The quark masses are $m_q = 0.34$ GeV for up and down quarks and $m_s = 0.55$ GeV for strange quarks. Since the outgoing Kaons are being considered point like particles in this calculation, the constant α^2 is chosen to be 0.2 GeV^2 to take into account part of the finite size effect, this is somewhat larger than that in the calculation of the electromagnetic transition[1, 19]. The masses and the decay widths for the S-channel resonances are taken from the recent particle data group[25].

We present our calculation of the differential cross sections for $\gamma p \rightarrow K^+\Lambda$ at $E_{lab} = 1.2$ in Fig. 1 and at $E_{lab} = 1.4$ GeV in Fig. 2. The coupling constants in this calculation are $\alpha_{KN\Lambda} \approx 4.0$ and $g_{K^*} \approx 0.4$. The calculations of the differential cross sections have been carried out from the threshold up to $E_{lab} = 2.0$ GeV, the results are in good agreement with the experimental data[26, 27]. There are some interesting features that can be learnt here; the differential cross section shows a strong forward peaking around $E_{lab} =$ 1.2 GeV. As the energy increases, the high partial wave resonances become increasingly important, this leads to a backward peaking at high energies, which are dominated by the high partial wave resonances. Another important feature in the quark model calculations is the form factors from the integrations of the spatial wavefunction as well as the Lorentz boost factors; the total cross section as a function of energies is shown in Fig. 3, the result is consistent with the known data up to energy around 2 GeV in the laboratory frame. This has been a serious problem in the traditional hadronic models[7, 6]

The processes $\gamma p \rightarrow K^+\Sigma^0$ and $\gamma p \rightarrow K^0\Sigma^+$ can be calculated simultaneously in this approach; the difference between the charged and neutral Kaon productions for the Σ final states is the absence of the seagull term and the Kaon exchange in the T-channel, in the same time, the coupling constants g_{K^*} in Eq. 30 should have opposite signs with approximately equal magnitudes in the two processes due to the magnetic dipole transitions between K^* and K[28]. The U-channel and the S-channel resonance contributions in the

charged and neutral Kaon productions for the Σ final states can be related to each other by the isospin couplings. In Fig. 4 and 5, we show that the differential cross sections for $\gamma p \to K^+\Sigma^0$ at $E_{lab} = 1.157$ and 1.45 GeV, where the data come from the Bonn group[27, 29] and Ref. [30]. The resulting coupling constants are $\alpha_{K\Sigma P} = 2.4$ and $g_{K^*} = -2.5$. The isospin 3/2 resonances, in particular those belonging to **56** multiplet, play a dominant role for the Σ final states. In the $E_{lab} = 1.45$ GeV, the isospin 3/2 resonances $P_{31}(1910)$, $P_{33}(1920)$, $F_{35}(1950)$, and $F_{37}(1905)$ contribute significantly, and the F-wave resonances lead to a larger differential cross section in the backward angle. In the Fig. 6, we show our results of the differential cross section at $E_{lab} = 1.15$ and 1.4 GeV for $\gamma p \to K^0\Sigma^+$. Noticed that the seagull and s-channel nucleon pole terms have opposite signs so that they tend to cancel each other for $\gamma p \to K^+\Sigma^0$, while the cancelation does not exist for the $\gamma p \to K^0\Sigma^+$ because of the absence of the Seagull term. This leads a larger total cross section in $\gamma p \to K^0\Sigma^+$, and in the same time, the differential cross sections become less forward peaked. In Fig. 7, we show that total cross section for both $\gamma p \to K^+\Sigma^0$ and $K^0\Sigma^+$ comparing to the $\gamma p \to K^+\Sigma^0$ data[27]. Although the cross section for $\gamma p \to K^0\Sigma^+$ is larger than that for $\gamma p \to K^+\Sigma^0$, this represents much more improved calculations than the most hadronic models, which predict 10-100 times larger cross section for the K^0 production than that for the K^+ production[8]. The available data[31] for neutral Kaon production, $\gamma p \to K^0\Sigma^+$, are too poor to test the theory. However, one could compare the Kaon photoproduction for the Σ final states to the π photoproduction in the resonance $P_{33}(1232)$ region; in both cases, the isospin 3/2 resonance dominant, therefore, the underline dynamics for the two reactions are similar. The cancelation among the Born terms in the presence of the Seagull term leads to a smaller cross section around the peak of the resonance $P_{33}(1232)$ for the charge pion production, $\gamma p \to \pi^+ n$, although it dominates near the threshold, while there is no cancelation for the neutral pion production, $\gamma p \to \pi^0 p$[9]. This shows that the neutral Kaon production data is crucial to provide us insights into the dynamics of the meson-photoproductions.

No investigation of the Kaon as well as other meson photoproduction would be complete without the calculation of the polarizations. This can be easily carried in our approach, because the expressions of every polarization in terms of the CGLN amplitudes have been given in Refs. [5] and [7]. Here we only present our results for the recoil asymmetry P defined as

$$P = \frac{\frac{d\sigma}{d\Omega}^{\uparrow} - \frac{d\sigma}{d\Omega}^{\downarrow}}{\frac{d\sigma}{d\Omega}^{\uparrow} + \frac{d\sigma}{d\Omega}^{\downarrow}} \tag{65}$$

where $\uparrow$ ($\downarrow$) corresponds to the spin of the final state parallel (antiparallel) to the direction of the vector $\mathbf{k} \times \mathbf{q}$. The recoil asymmetry P is very important since it provides us a direct probe to the structure of the S-channel resonances in our approach. As the decay width for a resonance become large, it is important to treat it as a function of the momentum of the final decay particles; the decay width reaches its peak as the center mass energy equals to its mass, and decreases quickly if the total energy of the system moves away from the mass of the resonances. Therefore, it is only necessary to introduce the decay width for the S-channel resonances whose masses are higher than the threshold of the Kaon photoproduction. This shows that the imaginary part of the transition amplitudes only depends on the structure of the S-channel resonances above threshold, and there could be an overall phase factor for these resonances. In Fig. 8, we show the recoil asymmetry P as a function of E_{lab} at 90 degrees for both $\gamma p \to K^+\Lambda$ and $K^+\Sigma^0$, the asymmetry for the $\gamma p \to K^+\Sigma^0$ is small and positive comparing to the negative $\gamma p \to K^+\Lambda$ polarization, and this of course is in agreement with the experimental data[27, 6]. The physics for the opposite signs between the Λ and Σ final states is the dominance of the isospin 3/2 resonances in $\gamma p \to K^+\Sigma^0$. Perhaps more interesting feature of this calculation is the changing sign of the recoil asymmetry for $\gamma p \to K^+\Lambda$ in the backward angle as the energy increases. In Fig. 9, we show the asymmetry P for $\gamma p \to K^+\Lambda$ as a function of the scattering angle at $E_{lab} = 1.157$ and 1.4 GeV, the asymmetry P is negative at $E_{lab} = 1.1$ GeV and becomes positive in the backward angle at 1.4 GeV. Indeed, the recent data from Bonn group[27] show some hints that this might be indeed the case. This feature is new and has not been shown in early calculations[7], the reason for the changing signs is the higher partial wave resonances become more important as the energy increases.

7. Future Work and Summary

Our calculations show that the chiral quark model presents a much better framework to understand the reaction mechanism of the Kaon photoproductions than many traditional hadronic models. In the same time, it also raises some interesting issues that need to be further studied. First, the relation between the chiral quark model and the Quark Pair Creation (QPC) Model[14] which has been quite successful phenomenologically in studying the strong decays of hadrons[32]. Thus, it is natural to calculate the meson-photoproduction in the QPC model, which has been done by Kumar and Onley[33]. The problem in their approach is that the QPC model alone does not lead to the low energy theorem near the threshold. This can be seen in the Seagul term that

dominates the low energy behaviour; the simple gauge transformation in the QPC leads to a Seagul term that is equivalent to the process $\gamma \to s\bar{s}$ inside nucleons. However, the chiral transformation tells us that there is another Seagul term that is equivalent to the process $\gamma u(d) \to u(d)$ for $K^+(K^0)$ production, which is not present in the simple QPC model. The relation between the two terms is determined by the chiral transformation; thus the total Seagul term has the structure;

$$\mathcal{M}_{sea} = e_{u(d)} R(\gamma u(d) \to u(d)) - e_s R(\gamma \to s\bar{s}) \tag{66}$$

where $e_{u(d)}$ is the charge of the up quark for the K^+ production and the down quarks for the K^0 production, e_s is the charge of strange quarks and R represents the spatial integral for each process. If the finite size of the emitting Kaon is being taken into account, the spatial integrals for the two processes are not equal;

$$R(\gamma u(d) \to u(d)) = N(\alpha^2, \beta^2) e^{-\frac{1}{1+\frac{2\alpha^2}{3\beta^2}} \left[\frac{1+\frac{\alpha^2}{\beta^2}}{6\alpha^2} (\mathbf{k}-\mathbf{q})^2 + \frac{\alpha^2}{18\beta^4} \left(\mathbf{k}^2 - \frac{1}{2}\mathbf{q}^2 \right) \right]} \tag{67}$$

and

$$R(\gamma \to s\bar{s}) = N(\alpha^2, \beta^2) e^{-\frac{1}{1+\frac{2\alpha^2}{3\beta^2}} \left[\frac{1}{6\alpha^2} (\mathbf{k}-\mathbf{q})^2 + \frac{5\alpha^2}{9\beta^2} \left(\frac{2}{3}\mathbf{k}^2 + \frac{1}{4} \left(5 + 4\frac{\alpha^2}{\beta^2} \right) \mathbf{q}^2 \right) \right]}, \tag{68}$$

where β^2 and α^2 are the harmonic oscillator strengths for the meson and baryon wavefunctions, and $N(\alpha^2, \beta^2)$ is the normalization factor, which is the same for both processes. The consequences for the finite size mesons are that there are additional terms proportional to on q^2 and k^2 which makes the the Seagull term decreases faster as the momentum $\mathbf{k}$ and $\mathbf{q}$ is increases, and it also generates an correction at order $\left(\frac{m_\pi}{M_N}\right)^2$ to the low energy theorem near the threshold[4] for both charge and neutral pion photoproductions. Moreover, a nonzero contribution from the Seagull term could affect the result of $\gamma p \to K^0\Sigma^+$ significantly at higher energies. This has not been discussed in the literature, and certainly deserves more study.

Second; the relativistic effects in the transition operator[34] should be included. Although the nonrelativistic transition operator used in this calculation is sufficient to reproduce the low energy theorem in threshold region, the relativistic corrections in the electromagnetic transition operator is important. In particular, the spin-orbital and so called nonadditive terms are required to derive the low energy theorem in the Compton scattering[11] and Drell Hearn Gerasimov sum rule[35], and they also breaks Moorhouse selection rule so that

the resonance $D_{13}(1700)$ would also contribute to the Kaon productions. Furthermore, the more consistent baryon wavefunction[20] should be used in the calculation.

Third; the role of gluonic degrees of freedom in the baryon wavefunction should be investigated. Our calculation shows a small contribution from the Roper resonance if it is a radial excitation of the nucleon, however, a hybrid Roper resonance would increase the contribution from the Roper resonance as well as the resonance $P_{33}(1600)$ significantly, the consequences of the hybrid Roper resonance and $P_{33}(1600)$ in the Kaon photoproductions remain to be studied.

Because these effects remain to be studied, the parameters, in particular the kaon coupling constants α_K, obtained in our calculations are by no means final, nor they are consistent with the measurements in Kaon-Nucleon scattering[36]. Our investigation does provide a framework to give a description of the Kaon-photoproductions that is consistent with the Kaon-nucleon scattering. Moreover, this approach makes it possible to provide a unified description for π, η and kaon photoproductions. The extension to the η and π photoproduction is in progress, and will be published elsewhere.

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Figure Caption

1. The differential cross section for $\gamma p \to K^+\Lambda$ at $E_{lab} = 1.2$ GeV, it has a $10^{-30} cm^2/sr$ unit.

2. The same as Fig. 1 at $E_{lab} = 1.4$ GeV.

3. The total cross section for $\gamma p \to K^+\Lambda$ as a function of the energy in the laboratory frame E_{lab}, it has a $10^{-30} cm^2$ unit.

4. The differential cross section for $\gamma p \to K^+\Sigma^0$ at $E_{lab} = 1.157$ GeV, it has a $10^{-30} cm^2/sr$ unit.

5. The same as Fig. 4 at $E_{lab} = 1.45$ GeV.

6. The differential cross sections for $\gamma p \to K^0\Sigma^+$, the solid and dashed lines correspond to $E_{lab} = 1.4$ and 1.15 GeV respectively.

7. The total cross sections as a function of E_{lab}, the data and solid line correspond to the process $\gamma p \to K^+\Sigma^0$, while the dashed line is the prediction for $\gamma p \to K^0\Sigma^+$.

8. The recoil asymmetry P at 90 degrees as a function of E_{lab}; the solid line and data correspond to the process $\gamma p \to K^+\Lambda$, and the dashed line represents $\gamma p \to K^+\Sigma^0$.

9. The asymmetry P as a function of scattering angle for $\gamma p \to K^+\Lambda$ at $E_{lab} = 1.1$ (solid line) and 1.4 (dashed line) GeV comparing to the data at $E_{lab} = 1.1$ GeV.

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